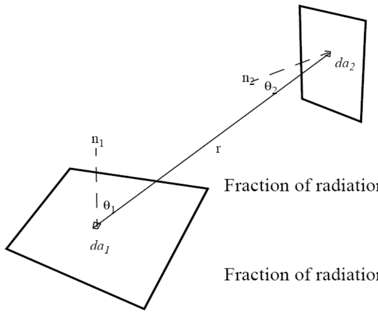
This section deals with the Analytical Solutions of Shape Factor available for some simplified geometries



$$F = \frac{1}{\pi} \cdot \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cdot \cos\theta_2 da_1 da_2}{r^2}$$

Fraction of radiation from A1 striking A2 $F_{12} = \frac{F}{A_1}$

Fraction of radiation from A2 striking A1 $F_{21} = \frac{F}{A_2}$

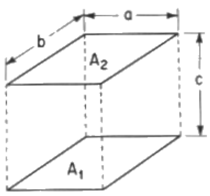
$F_{1\to2}$: This is the ratio of the rate at which surface 1 emits radiant energy which directly strikes surface 2 to the rate at which surface 1 emits radiant energy.

With emissivities e_1 and e_2, total transfer function is, allowing for multiple reflections

$$G = \frac{e_1 \cdot e_2 \cdot F}{1 - F_{12} \cdot F_{21} \cdot (1-e_1) \cdot (1-e_2)}$$

Given below is the summary of analytical calculation of view factors for some regular shape. However, this is just for the reference and we have not made any attempt to veryfy or derive the equations. Hence, the respective caculations are (copyright) of the original authors.

1 Finite Parallel Plates of Equal Size and Coplanar Edges



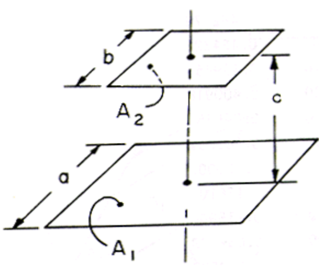
$$F_{1-2} = \frac{2}{\pi XY}\left\{ \ln\left[\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2}\right]^{1/2} + X\sqrt{1+Y^2}\tan^{-1}\frac{X}{\sqrt{1+Y^2}} + Y\sqrt{1+X^2}\tan^{-1}\frac{Y}{\sqrt{1+X^2}} - X\tan^{-1}X - Y\tan^{-1}Y \right\}$$

X = a/c Y = b/c

a	[mm]	100
b	[mm]	100
c	[mm]	100
X	[---]	1.000
Y	[---]	1.000
$F_{1\to2}$	[---]	0.1998

2 **Square** to **Square** in Parallel Plane: CG collinear

A = a/c B = b/a X = A.(1+B) Y = A.(1-B)



$$F_{1-2} = \frac{1}{\pi A^2}\left\{ \ln\frac{[A^2(1+B^2)+2]^2}{(Y^2+2)(X^2+2)} + (Y^2+4)^{1/2}\left[Y\tan^{-1}\frac{Y}{(Y^2+4)^{1/2}} - X\tan^{-1}\frac{X}{(Y^2+4)^{1/2}}\right] + (X^2+4)^{1/2}\left[X\tan^{-1}\frac{X}{(X^2+4)^{1/2}} - Y\tan^{-1}\frac{Y}{(X^2+4)^{1/2}}\right] \right\}$$

For A < 0.2 :

$$F_{1-2} = \frac{(AB)^2}{\pi}$$

a	[mm]	100
b	[mm]	100
c	[mm]	100
A	[mm]	1.00
B	[mm]	1.00
X	[---]	2.000
Y	[---]	0.000
K1	[---]	0.2877
K2	[---]	-3.1416
K3	[---]	3.4817
$F_{1\to2}$	[---]	0.1998

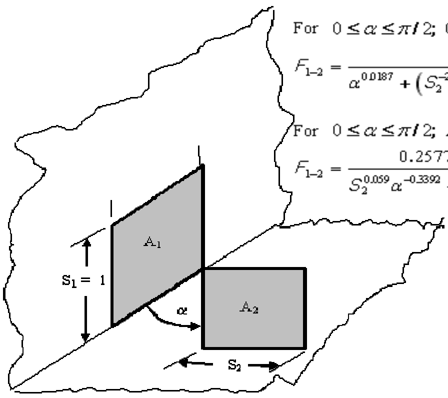
3

For $0 \le \alpha \le \pi/2;\ 0.1 \le S_2 \le 2.0$ (*max error* ±9.08%)

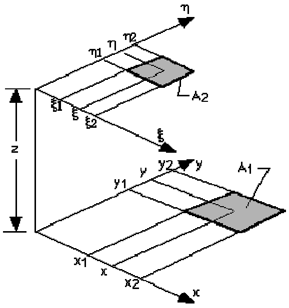
$$F_{1-2} = \frac{0.4399 S_2^{0.0871}}{\alpha^{0.0187} + \left(S_2^{-2.0849} + \alpha^{0.0871}\right)^{0.6387}\left(S_2^{1.5473} + \alpha^{1.4836}\right)^{1.5932}}$$

For $0 \le \alpha \le \pi/2;\ 2.0 \le S_2 \le 10.0$ (*max error* ±12.25%)

$$F_{1-2} = \frac{0.2577 S_2^{-1.6494}\alpha^{-1.8087}}{S_2^{0.059}\alpha^{-0.3392} + \left(1 + S_2\alpha^{-8.3298}\right)^{0.2152}}$$



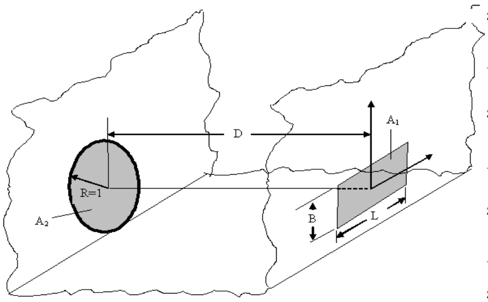
4 Rectangle to Rectangle in Parallel Planes: CG non-collinear



$$F_{1-2} = \frac{1}{(x_2 - x_1)(y_2 - y_1)} \sum_{l=1}^{2} \sum_{k=1}^{2} \sum_{j=1}^{2} \sum_{i=1}^{2} (-1)^{(i+j+k+l)} G(x_i, y_j, \eta_k, \xi_l)$$

$$G = \frac{1}{2\pi} \left(\begin{array}{l} (y-\eta)\left[(x-\xi)^2 + z^2\right]^{1/2} \tan^{-1}\left\{ \dfrac{y-\eta}{\left[(x-\xi)^2 + z^2\right]^{1/2}} \right\} \\ +(x-\xi)\left[(y-\eta)^2 + z^2\right]^{1/2} \tan^{-1}\left\{ \dfrac{x-\xi}{\left[(y-\eta)^2 + z^2\right]^{1/2}} \right\} \\ -\dfrac{z^2}{2} \ln\left[(x-\xi)^2 + (y-\eta)^2 + z^2\right] \end{array} \right)$$

5 Rectangle to Co-axial Disc in Parallel Plane



For $0.1 \le L \le 2.0;\ 0.1 \le B \le 2.0;\ 0.1 \le D \le 10.0$ (*max error* ±12.03%)

$$F_{1-2} = \frac{1.0152(1+L^{1.0251})(1+B^{3.4915})}{(1+L^{1.195})(1+B^{3.656})\left[D^{1.9767} + (1+D)^{0.3047} - 0.0175\right]}$$

$2.0 \le L \le 10.0;\ 2.0 \le B \le 10.0;\ 0.1 \le D \le 2.0$ (*max error* ±12.59%)

$$F_{1-2} = \frac{3.2718(1+D^{1.6491})^{0.2834}}{\left[D^{1.5138} + (BL)^{0.495}\right]^{2.0417}}$$

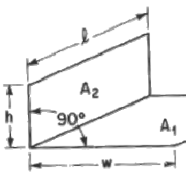
$2.0 \le L \le 10.0;\ 2.0 \le B \le 10.0;\ 2.0 \le D \le 10.0$ (*max error* ±11.42%)

$$F_{1-2} = \frac{1.1947(1+L^{0.4609})(1+B^{0.46})}{(1+L^{0.5111})(1+B^{0.5102})(D^{2.0405} + B^{1.195} + L^{1.1949} - 3.4734)^{2.0405}}$$

$0.1 \le L \le 2.0;\ 2.0 \le B \le 10.0;\ 0.1 \le D \le 10.0$ (*max error* ±18.64%)

$$F_{1-2} = 3.0932 D^{-0.1128}\left[BL + (BL)^2 + D + D^2 + D^4\right]^{-0.0188}\left(D^{1.0656} + L^{0.0215} B^{0.4893}\right)^{-2.2497}$$

6

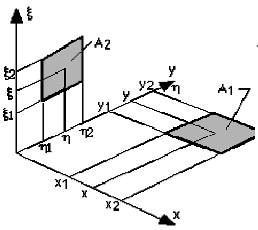


H = h/l W = w/l

$$F_{1-2} = \frac{1}{W\pi}\left(W \tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - \sqrt{H^2+W^2}\tan^{-1}\sqrt{\frac{1}{H^2+W^2}} + \frac{1}{4}\ln\left\{ \frac{(1+W^2)(1+H^2)}{1+W^2+H^2} \left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2} \right\}\right)$$

h	[mm]	100
l	[mm]	100
w	[mm]	100
H	[---]	1.00
W	[---]	1.00
K_1	[---]	0.70
K_2	[---]	1.33
K_3	[---]	0.750
K_4	[---]	0.750
K_5	[---]	-0.072
$F_{1\rightarrow 2}$	[---]	0.200

7 Rectangle to Rectangle in Perpendicular Planes
(All boundaries are either parallel or perpendicular to the (x,y) and (ξ,η) plane

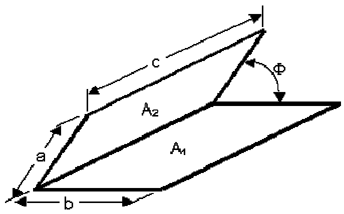


$$F_{1-2} = \frac{1}{(x_2 - x_1)(y_2 - y_1)} \sum_{l=1}^{2} \sum_{k=1}^{2} \sum_{j=1}^{2} \sum_{i=1}^{2} \left[(-1)^{(i+j+k+l)} G(x_i, y_j, \eta_k, \xi_l)\right]$$

where

$$G = \frac{1}{2\pi}\left\{ \begin{array}{l} (y-\eta)(x^2+\xi^2)^{1/2}\tan^{-1}\left[\dfrac{(y-\eta)}{(x^2+\xi^2)^{1/2}}\right] \\ -\dfrac{1}{4}\left[x^2+\xi^2-(y-\eta)^2\right]\ln\left[x^2+\xi^2+(y-\eta)^2\right] \end{array} \right\}$$

8 Two rectangles with one common edge and inclined angle ϕ



$$F_{1-2} = -\frac{\sin 2\Phi}{4\pi B}\left[AB\sin\Phi + \left(\frac{\pi}{2} - \Phi\right)(A^2 + B^2) + B^2\tan^{-1}\left(\frac{A - B\cos\Phi}{B\sin\Phi}\right) + A^2\tan^{-1}\left(\frac{B - A\cos\Phi}{A\sin\Phi}\right)\right]$$

$$+\frac{\sin^2\Phi}{4\pi B}\left\{\left(\frac{2}{\sin^2\Phi} - 1\right)\ln\left[\frac{(1+A^2)(1+B^2)}{1+C}\right] + B^2\ln\left[\frac{B^2(1+C)}{(1+B^2)C}\right] + A^2\ln\left[\frac{A^2(1+A^2)^{\cos 2\Phi}}{C(1+C)^{\cos 2\Phi}}\right]\right\}$$

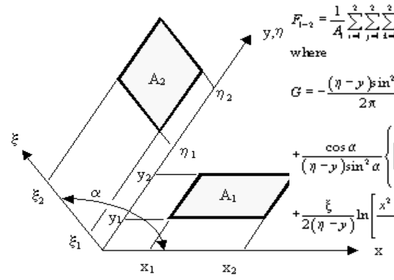
$$+\frac{1}{\pi}\tan^{-1}\left(\frac{1}{B}\right) + \frac{A}{\pi B}\tan^{-1}\left(\frac{1}{A}\right) - \frac{\sqrt{C}}{\pi B}\tan^{-1}\left(\frac{1}{\sqrt{C}}\right)$$

$$+\frac{\sin\Phi\sin 2\Phi}{2\pi B}AD\left[\tan^{-1}\left(\frac{A\cos\Phi}{D}\right) + \tan^{-1}\left(\frac{B - A\cos\Phi}{D}\right)\right]$$

$$+\frac{\cos\Phi}{\pi B}\int_0^B \sqrt{1+\xi^2\sin^2\Phi}\left[\tan^{-1}\left(\frac{\xi\cos\Phi}{\sqrt{1+\xi^2\sin^2\Phi}}\right) + \tan^{-1}\left(\frac{A - \xi\cos\Phi}{\sqrt{1+\xi^2\sin^2\Phi}}\right)\right]d\xi$$

$A = a/c$ $B = b/c$ $C = A^2 + B^2 - 2AB\cos(\phi)$ $D = \sqrt{A^2\sin^2(\phi) + 1}$

9 Two Rectangles in different planes inclined at angle α:



$$F_{1-2} = \frac{1}{A_1}\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2}\left[(-1)^{(i+j+k+l)}G(x_i, y_j, \eta_k, \xi_l)\right]$$

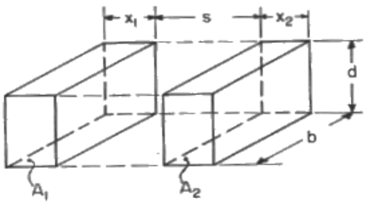
where

$$G = -\frac{(\eta - y)\sin^2\alpha}{2\pi}\int_{\xi}\left(\frac{(x-\xi\cos\alpha)\cos\alpha - \xi\sin^2\alpha}{\left(x^2 - 2x\xi\cos\alpha + \xi^2\right)^{1/2}\sin^2\alpha}\tan^{-1}\left[\frac{\eta - y}{\left(x^2 - 2x\xi\cos\alpha + \xi^2\right)^{1/2}}\right]\right.$$

$$+\frac{\cos\alpha}{(\eta - y)\sin^2\alpha}\left\{\left[\xi^2\sin^2\alpha + (\eta - y)^2\right]^{1/2}\tan^{-1}\frac{x - \xi\cos\alpha}{\left[\xi^2\sin^2\alpha + (\eta-y)^2\right]^{1/2}} - \xi\sin\alpha\tan^{-1}\left(\frac{x - \xi\cos\alpha}{\sin\alpha}\right)\right\}$$

$$\left.+\frac{\xi}{2(\eta - y)}\ln\left[\frac{x^2 - 2x\xi\cos\alpha + \xi^2 + (\eta - y)^2}{x^2 - 2x\xi\cos\alpha + \xi^2}\right]\right)d\xi$$

10 Finite area on Interior of Rectangular Enclosure to Second Finite Area

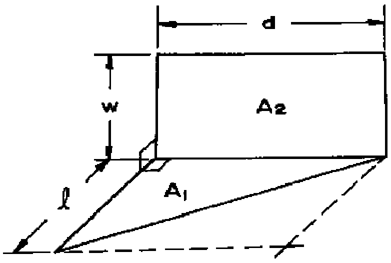


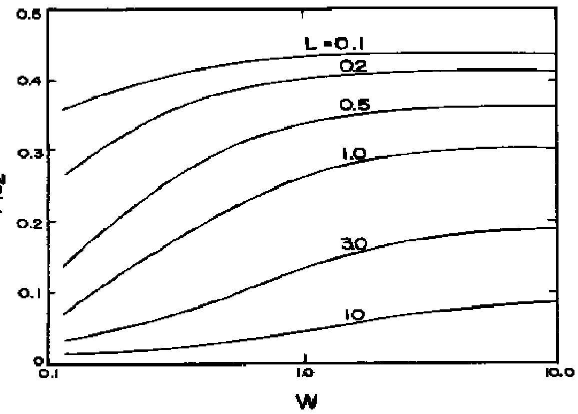
$$B = \frac{b}{d};\ X = \frac{x}{d};\ S = \frac{s}{d}$$

$$f(Z) = B\left(Z^2 + 1\right)^{1/2}\tan^{-1}\frac{B}{\left(Z^2+1\right)^{1/2}} - BZ\tan^{-1}\frac{B}{Z} + \left(Z^2 + B^2\right)^{1/2}\tan^{-1}\frac{1}{\left(Z^2 + B^2\right)^{1/2}}$$

$$-Z\tan^{-1}\frac{1}{Z} + \frac{Z^2}{2}\ln\frac{\left(Z^2 + B^2\right)\left(Z^2+1\right)}{Z^2\left(Z^2 + B^2 + 1\right)}$$

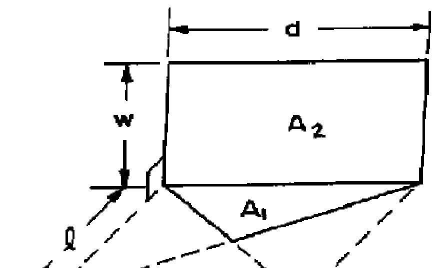
11 Right angled Triangle to Perpendicular Rectangle with Common Edge (ASHARE)

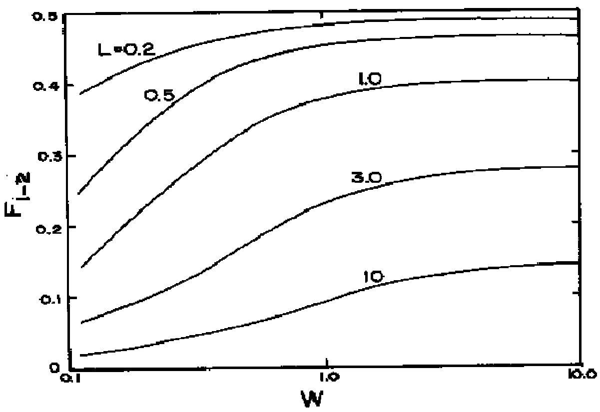




L = l/d W = w/d

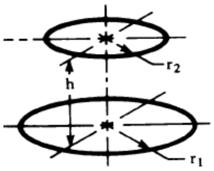
12 Isosceles Triangle to Perpendicular rectangle with One Common Edge: (ASHRAE)





L = l/d W = w/d

13 Finite Disc in Parallel Co-axial Disc of Unequal Radius

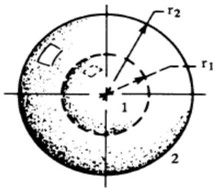


Let $R_1 = r_1/h$, $R_2 = r_2/h$, and $X = 1 + \left(1 + R_2^2\right)/R_1^2$.

$$F_{1\text{-}2} = \frac{1}{2}\left[X - \sqrt{X^2 - 4(R_2/R_1)^2}\right]$$

r_1	[m]	1.00
r_2	[m]	1.00
h	[m]	0.50
R_1	[---]	2.00
R_2	[---]	2.00
X	[---]	2.25
$F_{1\to2}$	[---]	0.61

14 Concentric Spheres



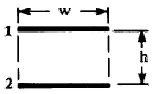
Concentric spheres:

$$F_{1\text{-}2} = 1, \quad F_{2\text{-}1} = (r_1/r_2)^2, \quad F_{2\text{-}2} = 1 - (r_1/r_2)^2$$

15 Infinite Geometries

Parallel Walls with Equal Width

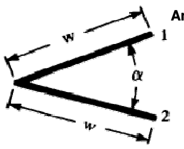
15a



$$F_{1\text{-}2} = F_{2\text{-}1} = \sqrt{1 + \left(\frac{h}{w}\right)^2} - \left(\frac{h}{w}\right)$$

Angular Co-edge Walls

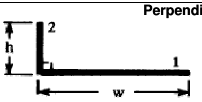
15b



$$F_{1\text{-}2} = F_{2\text{-}1} = 1 - \sin(\alpha/2)$$

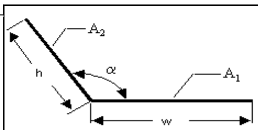
Perpendicular Co-edge Walls

15c



$$F_{1\text{-}2} = \frac{1}{2}\left[1 + \frac{h}{w} - \sqrt{1 + \left(\frac{h}{w}\right)^2}\right]$$

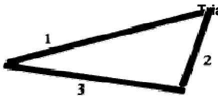
A = h/w



$$F_{1-2} = \frac{A + 1 - (A^2 + 1 - 2A\cos\alpha)^{1/2}}{2}$$

Triangular Enclosure: Plane or Convex Surfaces

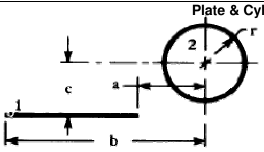
15d



$$F_{1\text{-}2} = (A_1 + A_2 - A_3)/2A_1$$

Plate & Cylinders

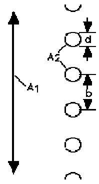
15e



$$F_{1\text{-}2} = \frac{r}{b-a}\left[\tan^{-1}\frac{b}{c} - \tan^{-1}\frac{a}{c}\right]$$

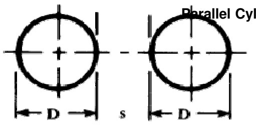
Infinite plane to row of parallel cylinders, or rows of in-line cylinders

D = d/b



Parallel Cylinders

15f

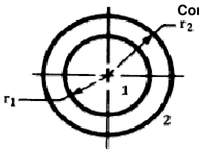


Let $X = 1 + s/D$. Then:

$$F_{1\text{-}2} = F_{2\text{-}1} = \frac{1}{\pi}\left[\sqrt{X^2 - 1} + \sin^{-1}\frac{1}{X} - X\right]$$

Concentric Cylinders

15g



$$F_{1\text{-}2} = 1, \quad F_{2\text{-}1} = \frac{r_1}{r_2}, \text{ and}$$

$$F_{2\text{-}2} = 1 - F_{2\text{-}1} = 1 - \frac{r_1}{r_2}$$

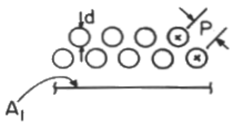
$$F_{1-2} = 1 - \left(1 - D^2\right)^{1/2} + D\tan^{-1}\left(\frac{1-D^2}{D^2}\right)^{1/2}$$

For n rows of in-line pipes:

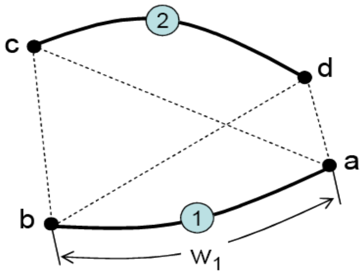
$$F_{1\text{-}n\_rows} = 1 - \left(1 - F_{1\text{-}2}\right)^n$$

Infinite plane to first, second, and first plus second rows of infinitely long parallel tubes of equal diameter in equilateral triangular array

R	$F_{1\text{-front row}}$	F1- 2nd row	R	$F_{1\text{-front row}}$	F1- 2nd row	R	$F_{1\text{-front row}}$	F1- 2nd row
1.5	0.8154	0.138	4.0	0.3613	0.2008	6.5	0.2298	0.1574
2.0	0.6576	0.1953	4.5	0.3243	0.1916	7.0	0.2142	0.1503
2.5	0.5472	0.214	5.0	0.2941	0.1824	8.0		
3.0	0.4675	0.2149	5.5	0.269	0.1735	9.0		
3.5	0.4077	0.2093	6.0	0.2479	0.1652	10.0		



16 2D surfaces, infinite in one extent CROSS-STRING METHOD



$$F_{12} = \frac{1}{2w_1}\left|(ac + bd) - (ad + bc)\right|$$

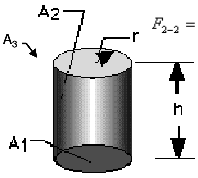
W_1 = Arc length along surface-1

17 View Factor of Cylinder Wall w.r.t. Individual Cap

H = h/2r

$$F_{1-2} = 2H\left[(1 + H^2)^{1/2} - H\right]$$

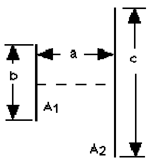
$$F_{2-2} = (1 + H) - (1 + H^2)^{1/2}$$



h	[m]	0.50
r	[m]	1.00
H	[---]	0.25
$F_{1\to2}$	[---]	0.39
$F_{2\to2}$	[---]	0.22
$F_{2\to1}$	[---]	0.39
$F_{2\to3}$	[---]	0.39
$F_{1\to3}$	[---]	0.61

18 Two infinitely long parallel plates of different widths: centerlines of plates are connected by perpendicular between plates.

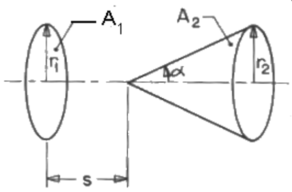
B = b/a C = c/a



$$F_{1-2} = \frac{1}{2B}\left[\sqrt{(B+C)^2 + 4} - \sqrt{(C-B)^2 + 4}\right]$$

a	[mm]	50
b	[mm]	200
c	[mm]	400
B	[mm]	4.00
C	[mm]	8.00
$F_{1\to2}$	[---]	0.48

19 Disc to Co-axial Cone:



$S = s/r_1$; $R = r_2/r_1$; $X = (S + R\cot\alpha)$

$A = [X^2 + (1 + R)^2]^{1/2}$; $B = [X^2 + (1 - R)^2]^{1/2}$

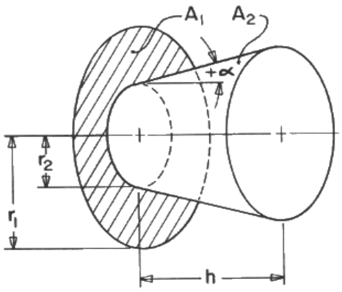
$C = (\cos\alpha + S\sin\alpha)^{1/2}$; $D = (\cos\alpha - S\sin\alpha)^{1/2}$

$E = R\cot\alpha - S$

$$F_{1-2} = \frac{1}{2}\left\{R^2 + X^2 + 1 - \left[(1 + R^2 + X^2)^2 - 4R^2\right]^{1/2}\right\}; \alpha \geq \tan^{-1}\frac{1}{S}$$

$$F_{1-2} = \frac{1}{\pi}\left\{-AB\tan^{-1}\frac{AC}{BD} + (1 + S^2)\tan^{-1}\frac{C}{D} + \frac{\sin\alpha}{\cos^2\alpha}\left[XE\tan^{-1}\frac{CD}{X} + S^2\tan^{-1}\frac{CD}{S} + (CD)^2\left(\tan^{-1}\frac{X}{CD} - \tan^{-1}\frac{S}{CD}\right)\right] + \left[\frac{R(X+S)}{\sin 2\alpha} - SR\tan\alpha\right]\cos^{-1}(-S\tan\alpha)\right\}; \alpha \leq \tan^{-1}\frac{1}{S}$$

20 Annular disk to coaxial truncated cone; cone can be convergent (+α) or divergent (-α).



$H = h/r_1$; $R = r_2/r_1$

$A = [H^2 + (1 + R + H\tan\alpha)^2]^{1/2}$

$B = [H^2 + (1 - R - H\tan\alpha)^2]^{1/2}$

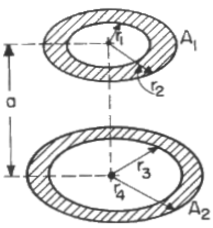
$C = (1 - R)^{1/2}$; $D = (1 + R)^{1/2}$; $E = \cos^2\alpha(1 - R^2)$

$$F_{1-2} = \frac{1}{\pi(1 - R^2)}\left\{-AB\tan^{-1}\frac{AC}{BD} + (CD)^2\tan^{-1}\frac{D}{C} + \frac{\sin\alpha}{\cos^2\alpha}\left[\left(H^2 + \frac{2HR}{\tan\alpha}\right)\tan^{-1}\frac{E^{1/2}}{H} + E\tan^{-1}\frac{H}{E^{1/2}}\right] + \left(\frac{H^2}{2\cos^2\alpha} + HR\tan\alpha\right)\cos^{-1}R\right\}$$

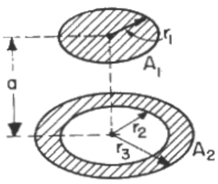
21 Ring to parallel coaxial ring

$H = a/r_1$; $R_2 = r_2/r_1$ Applicable to both the cases.

$R_3 = r_3/r_1$; $R_4 = r_4/r_1$



$$F_{1-2} = \frac{1}{2(R_2^2 - 1)}\left\{\left[(R_2^2 + R_3^2 + H^2)^2 - (2R_3R_2)^2\right]^{1/2} - \left[(R_2^2 + R_4^2 + H^2)^2 - (2R_2R_4)^2\right]^{1/2} + \left[(1 + R_4^2 + H^2)^2 - (2R_4)^2\right]^{1/2} - \left[(1 + R_3^2 + H^2)^2 - (2R_3)^2\right]^{1/2}\right\}$$

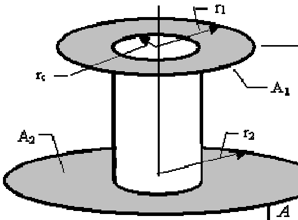


$$F_{1-2} = \frac{1}{2}\left\{R_3^2 - R_2^2 - \left[(1 + R_3^2 + H^2)^2 - 4R_3^2\right]^{1/2} + \left[(1 + R_2^2 + H^2)^2 - 4R_2^2\right]^{1/2}\right\}$$

22 Annulus to coaxial annulus of different outer radius; both annuli have inner radius of blocking coaxial cylinder

$R_1 = r_1/h$; $R_2 = r_2/h$; $R_c = r_c/h$; $A = R_1^2 - R_c^2$

$B = R_2^2 - R_c^2$; $C = R_2 + R_1$; $D = R_2 - R_1$; $Y = A^{1/2} + B^{1/2}$



$$F_{1-2} = \frac{1}{\pi A}\left[\begin{array}{l} \frac{A}{2}\cos^{-1}\frac{R_c}{R_2} + \frac{B}{2}\cos^{-1}\frac{R_c}{R_1} + 2R_c\left(\tan^{-1}Y - \tan^{-1}A^{1/2} - \tan^{-1}B^{1/2}\right) \\ -\left[\left(1+C^2\right)\left(1+D^2\right)\right]^{1/2}\tan^{-1}\left[\frac{\left(1+C^2\right)\left(Y^2-D^2\right)}{\left(1+D^2\right)\left(C^2-Y^2\right)}\right]^{1/2} \\ +\left\{\left[1+\left(R_1+R_c\right)^2\right]\left[1+\left(R_1-R_c\right)^2\right]\right\}^{1/2}\tan^{-1}\left\{\frac{\left[1+\left(R_1+R_c\right)^2\right]\left(R_1-R_c\right)}{\left[1+\left(R_1-R_c\right)^2\right]\left(R_1+R_c\right)}\right\}^{1/2} \\ +\left\{\left[1+\left(R_2+R_c\right)^2\right]\left[1+\left(R_2-R_c\right)^2\right]\right\}^{1/2}\tan^{-1}\left\{\frac{\left[1+\left(R_2+R_c\right)^2\right]\left(R_2-R_c\right)}{\left[1+\left(R_2-R_c\right)^2\right]\left(R_2+R_c\right)}\right\}^{1/2} \end{array}\right]$$

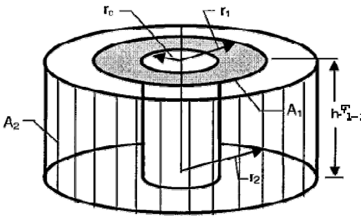
(use principal values in evaluating all inverse trig functions.)

r_1	[m]	1.000
r_2	[m]	1.000
r_c	[m]	0.500
h	[m]	0.500
R_1	[---]	2.000
R_2	[---]	2.000
R_c	[---]	1.000
A	[m^2]	3.000
B	[m^2]	3.000
C	[m]	4.000
D	[m]	0.000
Y	[---]	3.464
K_1	[---]	1.532
K_2	[---]	5.903
K_3	[---]	4.077
K_4	[---]	4.077
$F_{1\to2}$	[---]	0.402
A_1	[m^2]	2.356
A_2	[m^2]	2.356
$F_{2\to1}$	[---]	0.402

23 **Annular ring between two concentric cylinders to inside of outer cylinder, inner radius of ring is equal to radius of inner cylinder**

$R_1 = r_1/h$; $R_2 = r_2/h$; $R_c = r_c/h$; $A = R_1^2 - R_c^2$

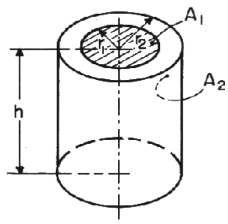
$B = R_2^2 - R_c^2$; $C = R_2 + R_1$; $D = R_2 - R_1$; $Y = A^{1/2} + B^{1/2}$



$$F_{1-2} = \frac{1}{\pi A}\left[\begin{array}{l} \frac{B}{2}\left(\pi - \cos^{-1}\frac{R_c}{R_2}\right) - 2R_c\left[\tan^{-1}Y - \tan^{-1}B^{1/2}\right] - \frac{1}{2}\cos^{-1}\frac{R_c}{R_1} \\ +\left[\left(1+C^2\right)\left(1+D^2\right)\right]^{1/2}\tan^{-1}\left[\frac{\left(1+C^2\right)\left(Y^2-D^2\right)}{\left(1+D^2\right)\left(C^2-Y^2\right)}\right]^{1/2} \\ -\left\{\left[1+\left(R_2+R_c\right)^2\right]\left[1+\left(R_2-R_c\right)^2\right]\right\}^{1/2}\tan^{-1}\left\{\frac{\left[1+\left(R_2+R_c\right)^2\right]\left(R_2-R_c\right)}{\left[1+\left(R_2-R_c\right)^2\right]\left(R_2+R_c\right)}\right\}^{1/2} \\ -\left(R_2^2-R_1^2\right)\tan^{-1}\left\{\frac{C}{D}\left[\frac{Y^2-D^2}{C^2-Y^2}\right]^{1/2}\right\} \end{array}\right]$$

r_c	[m]	0.500
r_1	[m]	0.750
r_2	[m]	1.000
h	[m]	0.500
R_1	[---]	1.500
R_2	[---]	2.000
R_c	[---]	1.000
A	[---]	1.250
B	[---]	3.000
C	[---]	3.500
D	[---]	0.500
Y	[---]	2.850
K_1	[---]	2.349
K_2	[---]	5.502
K_3	[---]	6.575
K_4	[---]	2.569
$F_{1\to2}$	[---]	-0.329

24 **Disk in cylinder base or top to inside surface of right circular cylinder** $R = r_2/r_1$; $H = h/r_1$

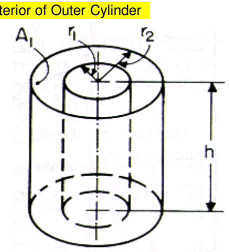


$$F_{1-2} = \frac{1}{2}\left\{1 - R^2 - H^2 + \left[\left(1+R^2+H^2\right)^2 - 4R^2\right]^{1/2}\right\}$$

r_1	[m]	0.500
r_2	[m]	1.000
h	[m]	0.500
R	[---]	2.000
H	[---]	1.000
$F_{1\to2}$	[---]	0.236

25 **Interior of finite length right circular coaxial cylinder to itself** $R_1 = r_1/h$; $R_2 = r_2/h$

Interior of Outer Cylinder

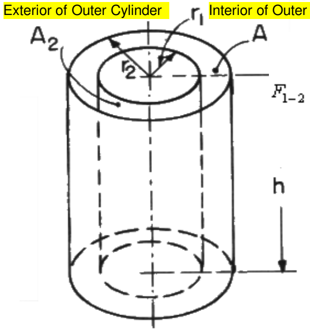


$$F_{1-1} = \frac{1}{\pi R_2}\left\{\begin{array}{l} \pi\left(R_2 - R_1\right) + \cos^{-1}\left(\frac{R_1}{R_2}\right) \\ -\left(1+4R_2^2\right)^{1/2}\tan^{-1}\frac{\left[\left(1+4R_2^2\right)\times\left(R_2^2-R_1^2\right)\right]^{1/2}}{R_1} \\ +2R_1\tan^{-1}\left[2\left(R_2^2-R_1^2\right)^{1/2}\right] \end{array}\right\}$$

(use principal values in evaluating all inverse trig functions)

r_1	[m]	0.500
r_2	[m]	1.000
h	[m]	0.500
R_1	[---]	1.000
R_2	[---]	2.000
K_1	[---]	4.189
K_2	[---]	5.903
K_3	[---]	2.580
$F_{1\to1}$	[---]	0.138

26 **Interior of outer right circular cylinder of finite length to exterior of coaxial inner right circular cylinder**



$R_1=r_1/h$; $R_2=r_2/h$; $A=R_2+R_1$; $B=R_2-R_1$

$$F_{1-2}=\frac{1}{\pi R_2}\left[\frac{1}{2}\left(R_2^2-R_1^2-1\right)\cos^{-1}\frac{R_1}{R_2}+\pi R_1-\frac{\pi}{2}AB-2R_1\tan^{-1}\left(R_2^2-R_1^2\right)^{1/2}+\left\{\left(1+A^2\right)\left(1+B^2\right)\right\}^{1/2}\tan^{-1}\left\{\frac{\left(1+A^2\right)B}{\left(1+B^2\right)A}\right\}^{1/2}\right]$$

Check: 1 OK

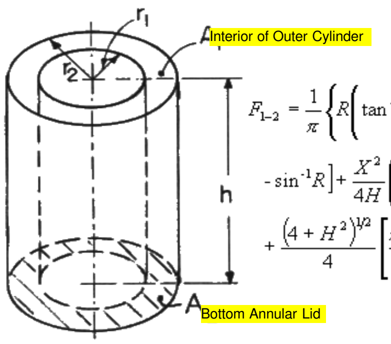
r_1	[m]	0.500
r_2	[m]	1.000
h	[m]	0.500
R_1	[---]	1.000
R_2	[---]	2.000
A	[---]	3.000
B	[---]	1.000
K_1	[---]	-0.524
K_2	[---]	2.094
K_3	[---]	4.077
$F_{1\to2}$	[---]	0.232
A_1	[m^2]	3.142
A_2	[m^2]	1.571
$F_{2\to1}$	[---]	0.465

Note: The symbols r_1, r_2 and A_1, A_2 are not consistent

27 **Interior of outer right circular cylinder of finite length to annular end enclosing space between coaxial cylinders**

$H = h/r_2$; $X = (1 - R^2)^{1/2}$

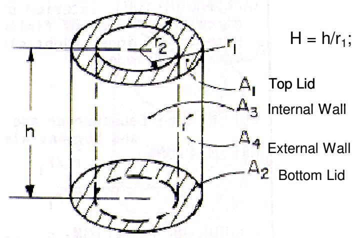
$R = r_1/r_2$; $Y = R(1 - R^2 - H^2)/(1 - R^2 + H^2)$



$$F_{1-2}=\frac{1}{\pi}\left\{R\left(\tan^{-1}\frac{X}{H}-\tan^{-1}\frac{2X}{H}\right)+\frac{H}{4}\left[\sin^{-1}\left(2R^2-1\right)-\sin^{-1}R\right]+\frac{X^2}{4H}\left(\frac{\pi}{2}+\sin^{-1}R\right)-\frac{\left[\left(1+R^2+H^2\right)^2-4R^2\right]^{1/2}}{4H}\left(\frac{\pi}{2}+\sin^{-1}Y\right)+\frac{\left(4+H^2\right)^{1/2}}{4}\left[\frac{\pi}{2}+\sin^{-1}\left(1-\frac{2R^2H^2}{4X^2+H^2}\right)\right]\right\}$$

r_1	[m]	0.500
r_2	[m]	1.000
h	[m]	0.500
H	[---]	0.500
R	[---]	0.500
X	[---]	0.866
Y	[---]	0.250
K_1	[---]	-0.121
K_2	[---]	0.654
K_3	[---]	1.019
K_4	[---]	1.476
$F_{1\to2}$	[---]	0.315
A_1	[m^2]	3.142
A_2	[m^2]	2.356
$F_{2\to1}$	[---]	0.420

28 **Annular end enclosing space between coaxial right circular cylinders to opposite annular end**



$H = h/r_1$; $R = r_2/r_1$

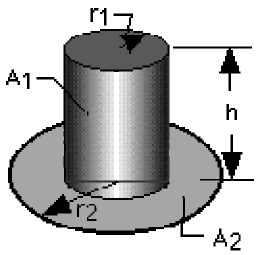
$$F_{1-2}=1-\left(\frac{H}{R^2-1}\right)\left[1-R\left(F_{4-4}+2F_{4-3}-1\right)\right]$$

	r_1	[m]	0.500
	r_2	[m]	1.000
	h	[m]	0.500
	H	[---]	1.000
	R	[---]	2.000
	$F_{4\to4}$	[---]	0.138
	$F_{4\to3}$	[---]	0.230
	$F_{1\to2}$	[---]	0.399
As per Forumula - 22	$F_{1\to2}$	[---]	0.402

29 **Outer surface of cylinder to annular disk at end of cylinder**

$R = r_1/r_2$; $H = h/r_2$

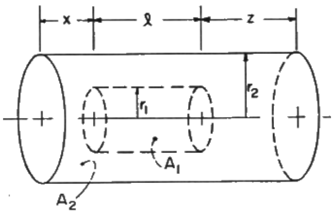
$A = H^2 + R^2 - 1$; $B = H^2 - R^2 + 1$



$$F_{1-2}=\frac{B}{8RH}+\frac{1}{2\pi}\left\{\cos^{-1}\left(\frac{A}{B}\right)-\frac{1}{2H}\left[\frac{(A+2)^2}{R^2}-4\right]^{1/2}\times\cos^{-1}\left(\frac{AR}{B}\right)-\frac{A}{2RH}\sin^{-1}R\right\}$$

r_1	[m]	0.500
r_2	[m]	1.000
h	[m]	0.500
R	[---]	0.500
H	[---]	0.500
A	[---]	-0.500
B	[---]	1.000
K_1	[---]	-0.316
K_2	[---]	-0.083
$F_{1\to2}$	[---]	0.268
Countercheck		0.268
A_1	[m^2]	1.571
A_2	[m^2]	2.356
$F_{2\to1}$	[---]	0.178

30 **Inner coaxial cylinder to outer coaxial cylinder; inner cylinder entirely within outer**



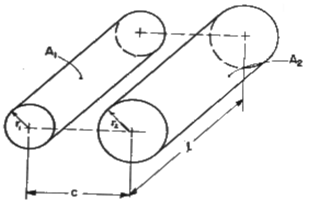
$X = x/r_2$; $Z = z/r_2$; $L = \ell/r_2$; $R = r_1/r_2$

$$F_{1-2}=1+\frac{X}{L}F_X+\frac{Z}{L}F_Z-\left(\frac{L+X}{L}\right)F_{L+X}-\frac{L+Z}{L}F_{L+Z}$$

$A_\xi = \xi^2 + R^2 - 1$; $B_\xi = \xi^2 - R^2 + 1$

$$F_\xi=\frac{B_\xi}{8R\xi}+\frac{1}{2\pi}\left\{\cos^{-1}\frac{A_\xi}{B_\xi}-\frac{1}{2\xi}\left[\frac{\left(A_\xi+2\right)^2}{R^2}-4\right]^{1/2}\cos^{-1}\frac{A_\xi R}{B_\xi}-\frac{A_\xi}{2\xi R}\sin^{-1}R\right\}$$

31 **Parallel opposed cylinders of unequal radius and equal finite length**



$$A=\frac{1}{2\pi R}\left\{\left[C^2-(1+R)^2\right]^{1/2}-\left[C^2-(1-R)^2\right]^{1/2}+\pi R+(1-R)\cos^{-1}\left(\frac{1-R}{C}\right)-(1+R)\cos^{-1}\left(\frac{1+R}{C}\right)\right\}$$

$$B=\frac{1}{\pi}\sin^{-1}\left(\frac{1}{C}\right)$$

$$C=1-\frac{1}{\pi}\left(\cos^{-1}\left(\frac{Y_1}{Z_1}\right)-\frac{1}{2RL}\left\{\left[\left(Y_1+2X_1^2\right)^2-\left(2X_1R\right)^2\right]^{1/2}\cos^{-1}\left(\frac{RY_1}{X_1Z_1}\right)+Y_1\sin^{-1}\left(\frac{R}{X_1}\right)-\frac{\pi}{2}Z_1\right\}\right)$$

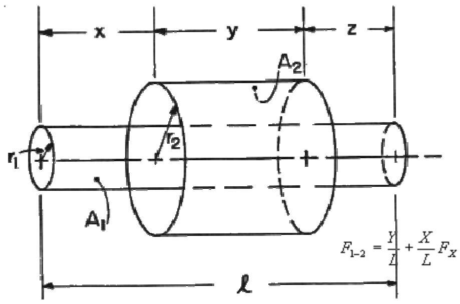
$$D=1-\frac{1}{\pi}\left(\cos^{-1}\left(\frac{Y_2}{Z_2}\right)-\frac{1}{2L}\left(Y_2R^4+2X_2^2\right)^2-\left(2X_2\right)^{1/2}\cos^{-1}\left(\frac{Y_2}{X_2Z_2}\right)+R^2Y_2\,\sin-1X_2-\left(\frac{\pi R^2Z^2}{2}\right)\right)$$

$$E=1-\frac{1}{\pi}\cos^{-1}\left(\frac{L^2-X_1^2}{L^2+X_1^2}\right)$$

$$X_1=\left[\frac{\left(C^2-1\right)^{1/2}-\left(\frac{\pi}{2}\right)}{\sin^{-1}\left(\frac{1}{C}\right)}+1\right]^{1/2};\quad X_2=R\left\{\frac{\left[\left(\frac{C}{R}\right)^2-1\right]^{1/2}-\frac{\pi}{2}}{\sin^{-1}\left(\frac{R}{C}\right)}+1\right\}^{1/2}$$

$$Y=L^2-X^2+R^2;\; Z=L^2+X^2-R^2$$

32 **Inner coaxial cylinder to outer coaxial cylinder; inner cylinder extends beyond both ends of outer**



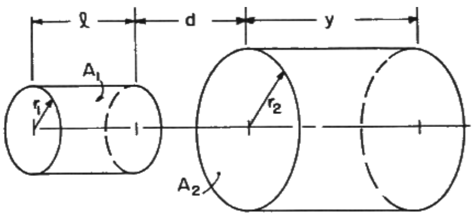
$$F_{1-2}=\frac{Y}{L}+\frac{X}{L}F_X+\frac{Z}{L}F_Z-\frac{X+Y}{L}F_{X+Y}-\frac{Z+Y}{L}F_{Y+Z}$$

$$X=x/r_2;\; Y=y/r_2;\; Z=z/r_2;\, L=\ell/r_2;\; R=r_1/r_2$$

$$A_\xi=\xi^2+R^2-1;\; B_\xi=\xi^2-R^2+1$$

$$F_\xi=\frac{B_\xi}{8R\xi}+\frac{1}{2\pi}\left\{\cos^{-1}\frac{A_\xi}{B_\xi}-\frac{1}{2\xi}\left[\frac{\left(A_\xi+2\right)^2}{R^2}-4\right]^{1/2}\cos^{-1}\frac{A_\xi R}{B_\xi}-\frac{A_\xi}{2\xi R}\sin^{-1}R\right\}$$

33 **Outside of inner (smaller) coaxial cylinder to inside of larger cylinder; smaller cylinder completely outside larger**

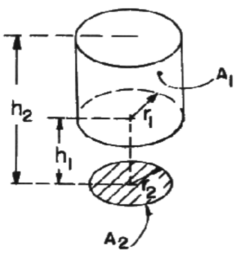


$$F_{1-2}=\frac{L+D}{L}F_{L+D}+\frac{Y+D}{L}F_{Y+D}-\frac{D}{L}F_D-\frac{L+D+Y}{L}F_{L+D+Y}$$

$$D=d/r_2;\; Y=y/r_2;\; L=\ell/r_2;\; R=r_1/r_2$$

$$A_\xi=\xi^2+R^2-1;\; B_\xi=\xi^2-R^2+1$$

$$F_\xi=\frac{B_\xi}{8R\xi}+\frac{1}{2\pi}\left\{\cos^{-1}\frac{A_\xi}{B_\xi}-\frac{1}{2\xi}\left[\frac{\left(A_\xi+2\right)^2}{R^2}-4\right]^{1/2}\cos^{-1}\frac{A_\xi R}{B_\xi}-\frac{A_\xi}{2\xi R}\sin^{-1}R\right\}$$



$$F_{1-2}=\frac{1}{4R\left(H_2-H_1\right)}\left[\left(X_1-X_2\right)-\left(X_1^2-4R^2\right)^{1/2}+\left(X_2^2-4R^2\right)^{1/2}\right]$$

$R = r_2/r_1$; $H_1 = h_1/r_1$, $H_2 = h_2/r_2$, $X = H^2 + R^2 + 1$