Material Properties

Viscosity: \( \mu_F := 1.975 \times 10^{-5} \text{ Pa}\cdot\text{s} \)

Mean Working Temperature: \( T_M := 25^\circ\text{C} \)

Volume flow: \( Q_F := 0.025 \text{ m}^3/\text{s} \)

Mean Absolute Pressure: \( p_m := 1.01325 \text{ bar} \)

Density: \( \rho_F := \frac{p_m}{287.1 \frac{J}{\text{kg}\cdot\text{K}}} = 1.184 \text{ kg/m}^3 \)

Surface Roughness

Ref. Diameter: \( D_0 := 20.0\text{mm} \) \( R_0 := 2.5\text{in} \)

Pipe Flow - Smooth and Rough:

\[
\begin{align*}
\epsilon_{D}(\rho, V, D_H, \mu, \epsilon) &= nRe - \frac{\rho \cdot V \cdot D_H}{\mu} \\
\Delta_{\text{LIM}} &= \frac{181 \cdot \log(nRe) - 16.4}{nRe} \\
&= 64.0 \quad \text{if} \quad nRe \leq 2300.0 \\
&= 0.04 \quad \text{if} \quad 2300 < nRe \leq 4000 \\
&= \frac{1}{(1.8 \cdot \log(nRe) - 1.64)^2} \quad \text{if} \quad 4000 < nRe < \Delta_{\text{LIM}} > \frac{\epsilon}{D_H} \\
&= 0.25 \quad \text{if} \quad 4000 < nRe \\
&= \left(\log\left(\frac{\epsilon}{D_H \cdot 3.7} + \frac{5.74}{nRe \cdot 0.9}\right)^2\right) \\
\end{align*}
\]

\( nRe := \text{Rey}(\rho_F, V_F, D_0, \mu_F) = 9.5 \times 10^4 \)

Check: \( f_{\epsilon_{D}}(\rho_F, V_F, D_0, \mu_F, \epsilon_R) = 0.02173 \quad \frac{\epsilon_R}{D_0} = 0.0008 \)

Filonenko & Altshul’s Formula

Colebrook-White-based formula (Roberson & Crowe, 1997)

\[
\text{Re}_{\text{Lm}} \left(\frac{\epsilon_R}{D_0}\right) = 1.8 \times 10 \\
\text{Re} := \frac{\rho V D}{\mu} \\
\]
\[
f = \frac{1}{2 \left( 2.51 \frac{\varepsilon_r}{nRe} \right)^2 + \left( 3.7D_0 \right)^2} \text{ solve } f \rightarrow 0.02159
\]

For "stabilized flow" in the region of "purely turbulent flow" and commercial circular tubes with non-uniform roughness of walls.

Note: This a transcendental non-linear equation that has to be solved! The Moody chart is just the graphical representation of this formulae. **Accuracy:** +/- 15%

\[
\lambda_x := 0.11 \left( 2 \frac{\varepsilon_r}{D_0} + \frac{68}{nRe} \right)^{0.25} = 0.0217
\]

**Ratio of Average to Centreline Velocity - Duct Flows**

Here, \( U_C \) is the centerline velocity.

\[
Q = U_C \int \left( 1 - \frac{r}{R} \right)^n dr = \frac{2\pi \cdot U_C \cdot n^2 \cdot R^2}{(n+1) \cdot (2n+1)} \text{ where } Q = \text{Volume Flow}
\]

\[
Q = \pi R^2 \cdot U_C = \frac{2\pi \cdot U_C \cdot n^2 \cdot R^2}{(n+1) \cdot (2n+1)} \Rightarrow U_C = \frac{Q}{\pi R^2} = \frac{(n+1) \cdot (2n+1)}{2n^2}
\]

\[
U(r) = U_C \left( 1 - \frac{r}{R} \right)^n \quad \text{where } V_r = \frac{(n+1) \cdot (2n+1)}{2n^2}
\]

\[
V_t(n) := \frac{(n+1) \cdot (2n+1)}{2n^2}
\]

\[
U_{bar} := 20 \text{ m/s} \quad U_C := U_{bar} \cdot V_t(7) = 24.5 \text{ m/s}
\]

**Exit from Tube - Circular and Square Tubes**

Check the Function

\[
\xi_{EXIT}(\text{Geom,n}) := \begin{cases} 
\frac{(2n+1)^3 \cdot (n+1)^3}{4n^4 \cdot (2n+3) \cdot (n+3)} & \text{if } n > 0 \land \text{Geom = "Circ"} \\
\frac{(n+1)^3}{n^2 \cdot (n+3)} & \text{if } n > 0 \land \text{Geom = "Plane"} \\
1.0 & \text{otherwise}
\end{cases}
\]

\[
\xi_{E1} := \xi_{EXIT}("\text{Plane",7}) = 1.045 \\
\xi_{E2} := \xi_{EXIT}("\text{Circ",7}) = 1.058
\]

**Sudden Expansion Tube - Circular and Square Tubes**

Note: Order of entering cross-section as IS NOT important.

\[
\xi_{EXPN}(\text{Geom,A0,A2,n}) := \begin{cases} 
M := \frac{(2n+1)^2 \cdot (n+1)}{4n^2 \cdot (n+2)} & \text{if } n > 0 \land \text{Geom = "Circ"} \\
N := \xi_{EXIT}(\text{Geom,n}) & \text{if } n > 0 \land \text{Geom = "Plane"} \\
\left( \min(A_0,A_2) \right)^2 + N - 2M \cdot \frac{\min(A_0,A_2)}{\max(A_0,A_2)} & \text{if } n > 1 \\
1 - \frac{\min(A_0,A_2)}{\max(A_0,A_2)} & \text{otherwise}
\end{cases}
\]

\[
\xi_{E3} := \xi_{EXPN}("\text{Plane",0.5,1.0,7}) = 0.2 \\
\xi_{E4} := \xi_{EXPN}("\text{Circ",0.5,1.0,7}) = 0.28 \\
\xi_{E5} := \xi_{EXPN}("\text{Circ",0.5,1.0,0}) = 0.25
\]

**Sudden Contraction**

Check: User has freedom to enter areas in any order

\[
\xi_{CONT}(A_0,A_1) := 0.50 \left( 1.0 - \frac{\min(A_0,A_1)}{\max(A_0,A_1)} \right)^{0.75} \\
\xi_{CONT}(0.5,1.0) = 0.297 \quad \xi_{CONT}(1.0,0.5) = 0.297
\]
Bends of Circular or Square Cross-Section (Idelchik 3rd Ed, Page 357)

**Equations:**

\[
\xi_{B,\text{Loc}}(R_0, D_0, \delta) := \begin{cases} 
(0.9 \cdot \sin(\delta)) & \text{if } \delta \leq 70\text{deg} \\
1.0 & \text{if } 70\text{deg} < \delta < 100\text{deg} \\
0.7 + 0.35 \left(\frac{\delta}{90\text{deg}}\right) & \text{otherwise}
\end{cases}
\]

\[
B_1 := 0.21 \left(\frac{R_0}{D_0}\right)^{-0.5} \quad \text{if } \frac{R_0}{D_0} > 1.0
\]

\[
B_1 := 0.21 \left(\frac{R_0}{D_0}\right)^{-2.5} \quad \text{otherwise}
\]

\[
A_1 = B_1 \cdot 1.0
\]

**Note:** The order in which \(R_0\) and \(D_0\) are entered is important.

**Walls assumed to be hydraulically smooth:** \(\varepsilon = 0\)

U-Shaped Joined Bends and Turns in one Plane - Idelchik 3rd Ed, Page 385

\[
\xi_{\text{BND}}(R_0, D_0, \delta, \rho, V, \mu) := \begin{cases} 
\xi_1 & \xi_{\text{B,Loc}}(R_0, D_0, \delta) \\
\lambda & \xi_{\varepsilon,D}(\rho, V, D_0, \mu, 0, 0) \\
\xi_1 + \frac{\pi \cdot \delta}{180\text{deg}} \cdot \frac{R_0}{D_0}\lambda & \text{otherwise}
\end{cases}
\]

Specify 'A' based on table on page 385
S-Bends of Circular or Square Cross-Section (IdelchiK 3rd Ed, Page 382)

\[
\begin{align*}
\mathbf{C}_\delta := \\
\begin{pmatrix}
2.032E-01 & 1.052E+00 & 1.793E+00 & 1.721E+00 \\
3.950E-01 & 1.905E-01 & 5.668E-01 & 3.096E-01 \\
6.042E-01 & 1.905E-01 & 4.687E-01 & 2.159E-02 \\
1.509E+00 & 2.216E-01 & 1.830E-01 & 1.261E-01 \\
1.052E+00 & 5.668E-01 & 2.548E-04 & 1.186E-06 \\
1.793E+00 & 3.096E-01 & 1.261E-01 & 3.701E-06 \\
1.721E+00 & 3.096E-01 & 1.261E-01 & 3.701E-06 \\
\end{pmatrix}
\begin{align*}
\delta_B := \\
\begin{pmatrix}
15 \\
30 \\
45 \\
60 \\
75 \\
90 \\
120 \\
\end{pmatrix}
\end{align*}
\]

\[
a_{15}(L_{EL}) := \sum_{j=0}^{6} \left( C_{\delta_{0,j}} L_{EL}^j \right) \\
a_{30}(L_{EL}) := \sum_{j=0}^{6} \left( C_{\delta_{1,j}} L_{EL}^j \right) \\
a_{45}(L_{EL}) := \sum_{j=0}^{6} \left( C_{\delta_{2,j}} L_{EL}^j \right) \\
a_{75}(L_{EL}) := \sum_{j=0}^{6} \left( C_{\delta_{4,j}} L_{EL}^j \right) \\
a_{90}(L_{EL}) := \sum_{j=0}^{6} \left( C_{\delta_{5,j}} L_{EL}^j \right)
\]

Check:

Fucntion ->
\[
\begin{align*}
a_{15}(0.0) &= 0.203 \\
a_{30}(1.0) &= 0.655 \\
a_{45}(2.0) &= 1.214 \\
a_{75}(3.0) &= 1.272 \\
a_{90}(3.0) &= 1.551
\end{align*}
\]

Idelchik Pg 382 ->
\[
\begin{align*}
a_{15}(0.0) &= 0.20 \\
a_{30}(1.0) &= 0.65 \\
a_{45}(2.0) &= 1.20 \\
a_{75}(3.0) &= 1.30 \\
a_{90}(3.0) &= 1.37
\end{align*}
\]

\[
\Delta% ->
\begin{align*}
0% & \quad 0% \\
1.17% & \quad -2.2% \\
13% & \quad
\end{align*}
\]

\[
A_{S\_BEND}(\delta, L_{EL}) := a_{120} \left( \sum_{j=0}^{7} C_{\delta_{6,j}} L_{EL}^j \right)
\]

(This function has not been used in this calculation)

Elbow with Sharp Corners - Idelchik 3rd Ed, Page 366

\[
\xi_{L\_SHARP}(\delta) := \left( 0.95 + \frac{33.5\text{deg}}{\delta} \right) \left( 0.95 \sin \left( \frac{\delta}{2} \right)^2 + 2.05 \sin \left( \frac{\delta}{2} \right)^4 \right)
\]
Interpolation function for Double Curved Turns (S-Shaped Bends)

\[ I_p(x_1, x_2, y_1, y_2, z_{11}, z_{12}, z_{21}, z_{22}) := \]
\[ z_{x1} \leftarrow z_{11} + \frac{x - x_1}{x_2 - x_1}(z_{21} - z_{11}) \]
\[ z_{x2} \leftarrow z_{12} + \frac{x - x_1}{x_2 - x_1}(z_{22} - z_{12}) \]
\[ z_{x1} + \frac{y - y_1}{y_2 - y_1}(z_{x2} - z_{x1}) \]

\[ A_S := I_p(22.5, 15.0, 30.0, 2.5, 2.0, 3.0, 0.60, 0.78, 0.88, 1.16) = 0.855 \quad \text{(Sample only)} \]

\[ \xi_{\text{BND}}(R_0, D_0, \delta, \rho, V, Dh, \mu, L_{\text{EL}}, A_S) := \]
\[ \lambda \leftarrow f_{E \_{\text{D}}}(\rho, V, Dh, \mu, 0.0) \]
\[ \xi_{\text{BND}}(R_0, D_0, \delta, \rho, V, \mu) \cdot A_S + \frac{\pi \cdot \delta}{180 \deg} \cdot R_0 \frac{\lambda}{D_0} + \frac{\lambda}{D_0} \]

90 deg Elbow - Made of 3 Elements at the angle 45 deg (Idelchik 3rd Ed, Page 374)

\[ \xi_{\text{EL}}(r, D) := \]
\[ AZ \leftarrow \begin{pmatrix} 1.118112 & -0.6977857 & -0.4818 & 0.70309 & -0.2244795 & -6.968263 \cdot 10^{-4} & 0.010588 & -1.241125 \cdot 10^{-3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \]
\[ \sum_{i=0}^{7} AZ_{0, i} \left( \frac{r}{D} \right)^i \]

Flow Exit from Side Wall of a Pipe with Recess

Side (Lateral) Exit from a Duct

In terms of velocity at MAIN cross-section.
Note: Order of entering cross-section area IS NOT important

\[ \xi = \frac{\Delta p}{\frac{1}{2} \rho w_0^2} \]

\[ \text{Side Exit} \left( A_{\text{ORF}}, A_0 \right) := \]
\[ \lambda \leftarrow \min \left( \frac{A_{\text{ORF}}}{A_0} \right) \]
\[ \max \left( \frac{A_{\text{ORF}}}{A_0} \right) \]
\[ 203.4 - 909.5 \cdot \lambda + 1105 \cdot \lambda^2 \quad \text{if} \quad \lambda \leq 0.40 \]
\[ 107.04 - 440.7 \cdot \lambda + 726.8 \cdot \lambda^2 - 539.87 \cdot \lambda^3 + 149.78 \cdot \lambda^4 \quad \text{if} \quad \lambda > 0.40 \]

Orifices - Thick Edge

In terms of velocity at ORIFICE cross-section
Note: Order of entering cross-section area IS important

\[ \xi = \frac{\Delta p}{\frac{1}{2} \rho V_0^2} \]
\[\xi_{ORF}(A_1, A_0, A_2, D_0, L, \lambda, \text{Type}) := \begin{cases} \eta_L \left\langle \frac{L}{D_0} \right. & \left. \begin{cases} - \left( \frac{0.535 \cdot \eta_L^8}{0.25 + \eta_L^8} \right) \quad \text{if Type = "Thick"} \\
 + \left( \frac{3.45 \cdot \eta_L + 88.4 \cdot \eta_L^{2.3}}{0.13 + 0.34 \cdot 10} \right) \quad \text{if Type = "Bevel"} \\
 0.50 & \text{if Type = "Sharp"} \end{cases} \right) \\
 \tau_0 \left\langle \begin{cases} 2.4 - \eta_L \cdot 10 & \text{if Type = "Thick"} \\
 \sqrt{2} & \text{if Type = "Bevel"} \\
 \eta_p & \text{if Type = "Sharp"} \end{cases} \right) \\
 \xi_0 \left\langle \frac{1}{\sqrt{2}} \right. & \left. \begin{cases} - \left( \frac{3.45 \frac{L}{D_0} + 88.4 \left( \frac{L}{D_0} \right)^{2.3}}{0.13 + 0.34 \cdot 10} \right) \quad \text{if Type = "Thick"} \\
 \sqrt{2} & \text{if Type = "Bevel"} \\
 \eta_p & \text{if Type = "Sharp"} \end{cases} \right) \\
 \xi_{SHARP}(A_1, A_0, A_2) := \left[ \frac{1}{2} \left( 1 - \frac{A_0}{A_1} \right) ^{0.75} + \left( 1 - \frac{A_0}{A_2} \right) ^{0.75} \right] ^{0.375} + \left( \frac{A_0}{A_1} \right) ^{2} \right) \\
 \xi_{ORF_BVL}(A_1, A_0, A_2, D_0, L, \lambda) := \begin{cases} \eta_p \left\langle \begin{cases} 0.13 + 0.34 \cdot 10 & \text{if Type = "Thick"} \\
 \frac{\sqrt{2}}{2} & \text{if Type = "Bevel"} \\
 \eta_p & \text{if Type = "Sharp"} \end{cases} \right) \\
 \tau_0 \left\langle \begin{cases} 2.4 - \eta_L \cdot 10 & \text{if Type = "Thick"} \\
 \sqrt{2} & \text{if Type = "Bevel"} \\
 \eta_p & \text{if Type = "Sharp"} \end{cases} \right) \\
 \xi_0 \left\langle \frac{1}{\sqrt{2}} \right. & \left. \begin{cases} - \left( \frac{3.45 \frac{L}{D_0} + 88.4 \left( \frac{L}{D_0} \right)^{2.3}}{0.13 + 0.34 \cdot 10} \right) \quad \text{if Type = "Thick"} \\
 \sqrt{2} & \text{if Type = "Bevel"} \\
 \eta_p & \text{if Type = "Sharp"} \end{cases} \right) \\
 \xi_{SHARP}(A_1, A_0, A_2) := \left[ \frac{1}{2} \left( 1 - \frac{A_0}{A_1} \right) ^{0.75} + \left( 1 - \frac{A_0}{A_2} \right) ^{0.75} \right] ^{0.375} + \left( \frac{A_0}{A_1} \right) ^{2} \right) \\
 \xi_{ORF_THK}(A_1, A_0, A_2, D_0, L, \lambda) := \begin{cases} \eta_L \left\langle \frac{L}{D_0} \right. & \left. \begin{cases} - \left( \frac{0.535 \cdot \eta_L^8}{0.25 + \eta_L^8} \right) \quad \text{if Type = "Thick"} \\
 + \left( \frac{3.45 \cdot \eta_L + 88.4 \cdot \eta_L^{2.3}}{0.13 + 0.34 \cdot 10} \right) \quad \text{if Type = "Bevel"} \end{cases} \right) \\
 \tau_0 \left\langle \begin{cases} 2.4 - \eta_L \cdot 10 & \text{if Type = "Thick"} \\
 \sqrt{2} & \text{if Type = "Bevel"} \\
 \eta_p & \text{if Type = "Sharp"} \end{cases} \right) \\
 \xi_0 \left\langle \frac{1}{\sqrt{2}} \right. & \left. \begin{cases} - \left( \frac{3.45 \frac{L}{D_0} + 88.4 \left( \frac{L}{D_0} \right)^{2.3}}{0.13 + 0.34 \cdot 10} \right) \quad \text{if Type = "Thick"} \\
 \sqrt{2} & \text{if Type = "Bevel"} \\
 \eta_p & \text{if Type = "Sharp"} \end{cases} \right) \\
 \xi_{SHARP}(A_1, A_0, A_2) := \left[ \frac{1}{2} \left( 1 - \frac{A_0}{A_1} \right) ^{0.75} + \left( 1 - \frac{A_0}{A_2} \right) ^{0.75} \right] ^{0.375} + \left( \frac{A_0}{A_1} \right) ^{2} \right) \\
 \xi_{EPB} := \xi_{ORF_THK}(49cm^2, 1.75cm^2, 49cm^2, D_H, 3.0mm, 0.02) = 2.08 \\
 \xi_{EPB2} := \xi_{ORF}(49cm^2, 1.75cm^2, 49cm^2, D_H, 3.0mm, 0.02, "Thick") = 2.08