Material Properties

Mean Working Temperature: $T_M := 25\,°C$

Viscosity: $\mu_F := 1.975 \cdot 10^{-5}\,Pa \cdot s$

Mean Absolute Pressure: $p_m := 1.01325\,bar$

Volume flow: $Q_F := 0.025\,\frac{m^3}{s}$

Density: $\rho_F := \frac{p_m}{287.1\,\frac{J}{kg \cdot K} \cdot T_M} = 1.184\,\frac{kg}{m^3}$

$\varepsilon_r := 0.016mm$
Surface Roughness

Ref. Diameter: $D_0 := 20.0mm$ $\quad R_0 := 2.5in$ $\quad F_0 := \pi \cdot \frac{{D_0}^2}{4} = 3.142 \times 10^{-4}\,m^2$ $\quad V_F := \frac{Q_F}{F_0} = 79.58\,\frac{m}{s}$

Pipe Flow - Smooth and Rough:

$$f_{\varepsilon\_D}(\rho, V, D_H, \mu, \varepsilon) := \left| \begin{array}{l} nRe \leftarrow \frac{\rho \cdot V \cdot D_H}{\mu} \\ \Delta_{LIM} \leftarrow \frac{181 \cdot \log(nRe) - 16.4}{nRe} \\ \frac{64.0}{nRe} \quad \text{if } nRe \le 2300.0 \\ 0.04 \quad \text{if } 2300 < nRe \le 4000 \\ \frac{1}{(1.8 \cdot \log(nRe) - 1.64)^2} \quad \text{if } 4000 < nRe \wedge \Delta_{LIM} > \frac{\varepsilon}{D_H} \\ \frac{0.25}{\left(\log\left(\frac{\varepsilon}{D_H \cdot 3.7} + \frac{5.74}{nRe^{0.9}}\right)\right)^2} \quad \text{if } 4000 < nRe \end{array} \right.$$

$$Rey(\rho, V, D, \mu) := \frac{\rho \cdot V \cdot D}{\mu}$$

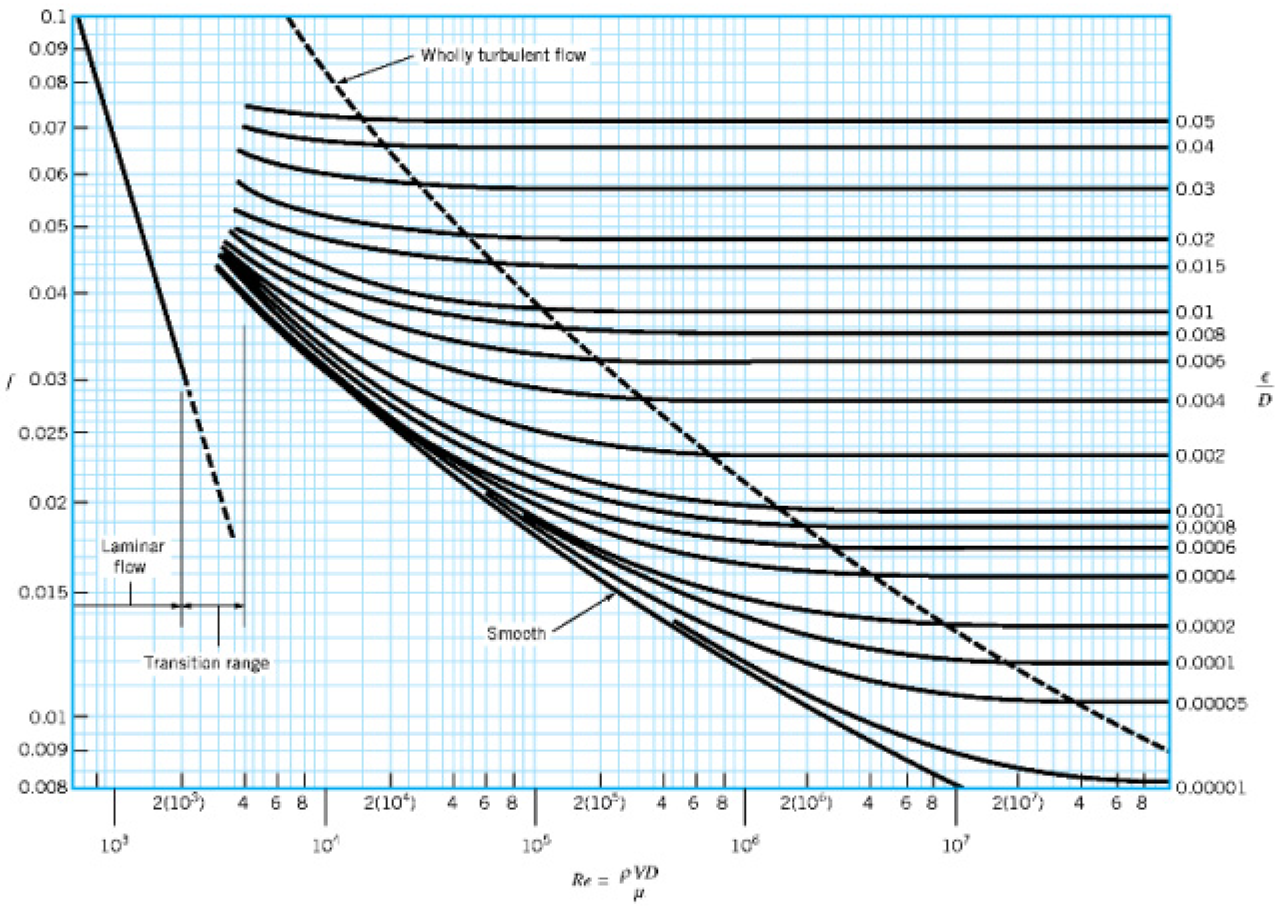
Filonenko & Altshul's Formula

Colebrook-White-based formula (Roberson & Crowe, 1997)

$$nRe := Rey(\rho_F, V_F, D_0, \mu_F) = 9.5 \times 10^4$$

$$Re_{Lm} \leftarrow \frac{217 - 382 \cdot \log\left(\frac{\varepsilon_r}{D_0}\right)}{\frac{\varepsilon_r}{D_0}} = 1.8 \times 10$$

Check: $f_{\varepsilon\_D}(\rho_F, V_F, D_0, \mu_F, \varepsilon_r) = 0.02173$ $\quad \frac{\varepsilon_r}{D_0} = 0.0008$



$$f = \frac{1}{\left(2 \cdot \log\left(\frac{2.51}{nRe \cdot \sqrt{f}} + \frac{\varepsilon_r}{3.7D_0}\right)\right)^2} \text{ solve}, f \rightarrow 0.02159$$

For "stabilized flow" in the region of "purely turbulent flow" and commercial circular tubes with non-uniform roughness of walls

Note: This a transcendental non-linear equation that has to be solved! The Moody chart is just the graphical representation of this formulae). **Accuracy:** +/- 15%

$$\lambda_x := 0.11 \cdot \left(\frac{\varepsilon_r}{D_0} + \frac{68}{nRe}\right)^{0.25} = 0.0217$$

Altshul's Approximation of Colebrook-White Implicit Formula

Ratio of Average to Centreline Veloicty - Duct Flows

$$U(r) = U_c\left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$ Here, U_C is the centerline velocity

$$V_r(n) := \frac{(n+1)\cdot(2\cdot n+1)}{2\cdot n^2}$$

$$Q = \int_0^R U_C\left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \cdot 2\pi r \cdot dr = 2\pi \cdot U_C \cdot \frac{n^2 \times R^2}{(n+1)\cdot(2n+1)} \text{ where } Q = \text{Volume Flow}$$

$$V_r(7) = 1.224$$

$$Q = \pi R^2 \times \bar{U} \quad \Rightarrow \bar{U} = U_C \cdot \frac{2n^2 \times R^2}{(n+1)\cdot(2n+1)} \quad \Rightarrow U_C = \bar{U} \cdot \frac{(n+1)\cdot(2n+1)}{2n^2}$$

$$V_r(8) = 1.195$$

$$\Rightarrow U(r) = \bar{U} \cdot \frac{(n+1)\cdot(2n+1)}{2n^2} \times \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} = U \cdot V_R \times \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \text{ where } V_R = \frac{(n+1)\cdot(2n+1)}{2n^2}$$

$$V_r(9) = 1.173$$

$$U_{bar} := 20\frac{m}{s} \qquad U_C := U_{bar} \cdot V_r(7) = 24.5\frac{m}{s}$$

$$V_r(10) = 1.155$$

Exit from Tube - Circular and Square Tubes

$$\xi_{EXIT}(Geom, n) := \begin{cases} \frac{(2\cdot n+1)^3 \cdot (n+1)^3}{4\cdot n^4 \cdot (2\cdot n+3)\cdot(n+3)} & \text{if } n > 0 \wedge Geom = \text{"Circ"} \\ \frac{(n+1)^3}{n^2 \cdot (n+3)} & \text{if } n > 0 \wedge Geom = \text{"Plane"} \\ 1.0 & \text{otherwise} \end{cases}$$

In terms of velocity in lower area

Check the Function

$$\xi_{E1} := \xi_{EXIT}(\text{"Plane"}, 7) = 1.045$$

$$\xi_{E2} := \xi_{EXIT}(\text{"Circ"}, 7) = 1.058$$

Sudden Expansion Tube - Circular and Square Tubes

*Note: Order of entering cross-section as **IS NOT** important*

$$\xi_{EXPN}(Geom, A_0, A_2, n) := \begin{array}{l} M \leftarrow \begin{cases} \frac{(2\cdot n+1)^2 \cdot (n+1)}{4\cdot n^2 \cdot (n+2)} & \text{if } n > 0 \wedge Geom = \text{"Circ"} \\ \frac{(n+1)^2}{n\cdot(n+2)} & \text{if } n > 0 \wedge Geom = \text{"Plane"} \end{cases} \\ N \leftarrow \xi_{EXIT}(Geom, n) \\ \left(\frac{\min(A_0, A_2)}{\max(A_0, A_2)}\right)^2 + N - 2M \cdot \frac{\min(A_0, A_2)}{\max(A_0, A_2)} \quad \text{if } n > 1 \\ \left(1 - \frac{\min(A_0, A_2)}{\max(A_0, A_2)}\right)^2 \quad \text{otherwise} \end{array}$$

Check the Function

$$\xi_{E3} := \xi_{EXPN}(\text{"Plane"}, 0.5, 1.0, 7) = 0.2$$

$$\xi_{E4} := \xi_{EXPN}(\text{"Circ"}, 0.5, 1.0, 7) = 0.28$$

$$\xi_{E5} := \xi_{EXPN}(\text{"Circ"}, 0.5, 1.0, 0) = 0.25$$

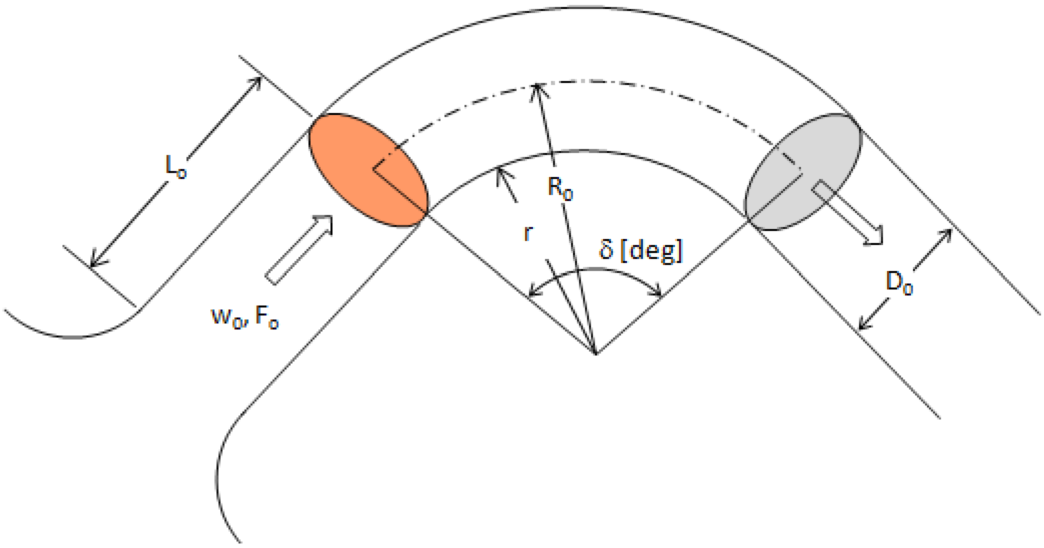
Sudden Contraction

$$\xi_{CONT}(A_0, A_1) := 0.50 \cdot \left(1.0 - \frac{\min(A_0, A_1)}{\max(A_0, A_1)}\right)^{0.75}$$

Check: User has freedom to enter areas in any order

$$\xi_{CONT}(0.5, 1.0) = 0.297 \qquad \xi_{CONT}(1.0, 0.5) = 0.297$$

Bends of Circular or Square Cross-Section (IdelchiK 3rd Ed, Page 357)



$$\xi_{B.Loc}(R_0, D_0, \delta) := \left| \begin{array}{l} A_1 \leftarrow \left| \begin{array}{ll} (0.9\cdot\sin(\delta)) & \text{if } \delta \le 70deg \\ 1.0 & \text{if } 70deg < \delta < 100deg \\ \left(0.7 + 0.35\cdot\dfrac{\delta}{90deg}\right) & \text{otherwise} \end{array} \right. \\ B_1 \leftarrow \left| \begin{array}{ll} 0.21\cdot\left(\dfrac{R_0}{D_0}\right)^{-0.5} & \text{if } \dfrac{R_0}{D_0} > 1.0 \\ 0.21\cdot\left(\dfrac{R_0}{D_0}\right)^{-2.5} & \text{otherwise} \end{array} \right. \\ A_1\cdot B_1\cdot 1.0 \end{array} \right.$$

*Note: The order in which R_0 and D_0 are entered **is important***

$$\xi_{BND}(R_0, D_0, \delta, \rho, V, \mu) := \left| \begin{array}{l} \xi_1 \leftarrow \xi_{B.Loc}(R_0, D_0, \delta) \\ \lambda \leftarrow f_{\varepsilon\_D}(\rho, V, D_0, \mu, 0.0) \\ \xi_1 + \dfrac{\pi\cdot\delta}{180deg}\cdot R_0\cdot\dfrac{\lambda}{D_0} \end{array} \right.$$

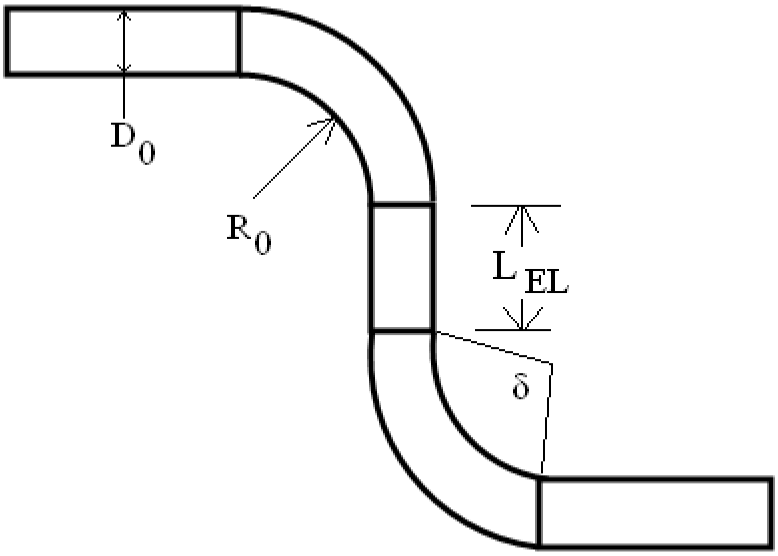
Walls assumed to be hydraulically smooth: $\varepsilon = 0$

U-Shaped Joined Bends and Turns in one Plane - Idelchik 3rd Ed, Page 385

$$\xi_{U.BND}(R_0, D_0, \delta, L_{EL}, A, \rho, V, \mu) := \left| \begin{array}{l} \xi_1 \leftarrow \xi_{B.Loc}(R_0, D_0, \delta) \\ \lambda \leftarrow f_{\varepsilon\_D}(\rho, V, D_0, \mu, 0.0) \\ A\cdot\xi_1 + \dfrac{\pi\cdot\delta}{180deg}\cdot R_0\cdot\dfrac{\lambda}{D_0} \end{array} \right.$$

Specify 'A' based on table on page 385

S-Bends of Circular or Square Cross-Section (IdelchiK 3rd Ed, Page 382)



$$C_\delta := \begin{pmatrix} 2.032E-01 & 2.136E-01 & -7.422E-03 & 1.501E-03 & -5.347E-04 & 4.586E-05 & -1.186E-06 \\ 3.950E-01 & 2.247E-01 & 5.838E-02 & -2.680E-02 & 3.999E-03 & -1.821E-04 & 3.599E-06 \\ 6.042E-01 & 6.302E-01 & -2.216E-01 & 3.457E-02 & -2.779E-03 & 1.148E-04 & -1.928E-06 \\ 1.509E+00 & 1.905E-01 & -1.830E-01 & 4.687E-02 & -5.230E-03 & 2.715E-04 & -5.359E-06 \\ 1.052E+00 & 5.668E-01 & -2.999E-01 & 5.982E-02 & -5.624E-03 & 2.548E-04 & -4.489E-06 \\ 1.793E+00 & -3.096E-01 & 1.261E-01 & -2.159E-02 & 1.905E-03 & -8.419E-05 & 1.469E-06 \\ 1.721E+00 & -8.899E-02 & -5.492E-02 & 2.355E-02 & -3.135E-03 & 1.780E-04 & -3.701E-06 \end{pmatrix} \quad \delta_B := \begin{pmatrix} 15 \\ 30 \\ 45 \\ 60 \\ 75 \\ 90 \\ 120 \end{pmatrix}$$

$$a_{15}(L_{EL}) := \sum_{j=0}^{6} \left(C_{\delta_{0,j}} \cdot L_{EL}^{\,j} \right) \qquad a_{30}(L_{EL}) := \sum_{j=0}^{6} \left(C_{\delta_{1,j}} \cdot L_{EL}^{\,j} \right) \qquad a_{45}(L_{EL}) := \sum_{j=0}^{6} \left(C_{\delta_{2,j}} \cdot L_{EL}^{\,j} \right)$$

$$a_{60}(L_{EL}) := \sum_{j=0}^{6} \left(C_{\delta_{3,j}} \cdot L_{EL}^{\,j} \right) \qquad a_{75}(L_{EL}) := \sum_{j=0}^{6} \left(C_{\delta_{4,j}} \cdot L_{EL}^{\,j} \right) \qquad a_{90}(L_{EL}) := \sum_{j=0}^{6} \left(C_{\delta_{5,j}} \cdot L_{EL}^{\,j} \right)$$

$$a_{120}(L_{EL}) := \sum_{j=0}^{6} \left(C_{\delta_{7,j}} \cdot L_{EL}^{\,j} \right)$$

Check:

Fucntion ->	$a_{15}(0.0) = 0.203$	$a_{30}(1.0) = 0.655$	$a_{45}(2.0) = 1.214$	$a_{75}(3.0) = 1.272$	$a_{90}(3.0) = 1.551$
Idelchik Pg 382 ->	0.20	0.65	1.20	1.30	1.37
Δ% ->	0%	0%	1.17%	−2.2%	13%

$$A_{S\_BEND}(\delta, L_{EL}) := a_{120} \leftarrow \sum_{j=0}^{7} \left(C_{\delta_{6,j}} \cdot L_{EL}^{\,j} \right)$$

**(This function has not been used in this calculation)**

Elbow with Sharp Corners - Idelchik 3rd Ed, Page 366

$$\xi_{L\_SHARP}(\delta) := \left(0.95 + \frac{33.5deg}{\delta}\right) \cdot \left(0.95 \cdot \sin\left(\frac{\delta}{2}\right)^2 + 2.05 \cdot \sin\left(\frac{\delta}{2}\right)^4\right)$$

Interpolation function for Double Curved Turns (S-Shaped Bends)

$$I_p(x,x_1,x_2,y,y_1,y_2,z_{11},z_{12},z_{21},z_{22}) := \left| \begin{array}{l} z_{x1} \leftarrow z_{11} + \dfrac{x-x_1}{x_2-x_1}\cdot(z_{21}-z_{11}) \\ z_{x2} \leftarrow z_{12} + \dfrac{x-x_1}{x_2-x_1}\cdot(z_{22}-z_{12}) \\ z_{x1} + \dfrac{y-y_1}{y_2-y_1}\cdot(z_{x2}-z_{x1}) \end{array} \right.$$

$A_S := I_p(22.5,15.0,30.0,2.5,2.0,3.0,0.60,0.78,0.88,1.16) = 0.855$ ***(Sample only)***

$$\xi_{sBND}(R_0,D_0,\delta,\rho,V,Dh,\mu,L_{EL},A_S) := \left| \begin{array}{l} \lambda \leftarrow f_{\varepsilon\_D}(\rho,V,Dh,\mu,0.0) \\ \xi_{BND}(R_0,D_0,\delta,\rho,V,\mu)\cdot A_S + \dfrac{\pi\cdot\delta}{180deg}\cdot R_0\cdot\dfrac{\lambda}{D_0} + \lambda\cdot\dfrac{L_{EL}}{D_0} \end{array} \right.$$

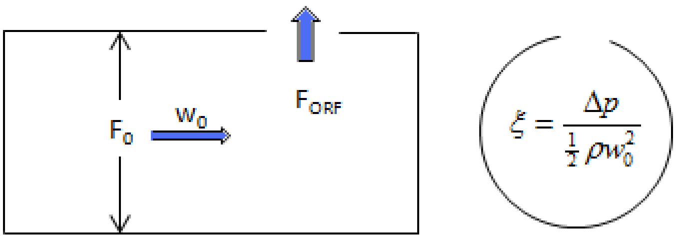
90 deg Elbow - Made of 3 Elements at the angle 45 deg (Idelchik 3rd Ed, Page 374)

$$\xi_{EL}(r,D) := \left| \begin{array}{l} AZ \leftarrow \begin{pmatrix} 1.118112 & -0.6977857 & -0.4818 & 0.70309 & -0.2244795 & -6.968263\cdot10^{-4} & 0.010588 & -1.241125\cdot10^{-3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ \left[\displaystyle\sum_{i=0}^{7}\left[AZ_{0,i}\cdot\left(\frac{r}{D}\right)^i\right]\right] \end{array} \right.$$

Flow Exit from Side Wall of a Pipe with Recess

Side (Lateral) Exit from a Duct

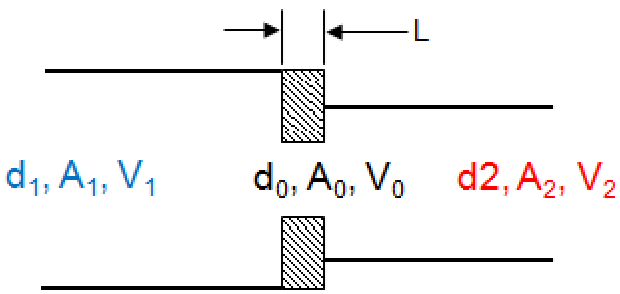
*In terms of velocity at **MAIN** cross-section.*
*Note: Order of entering cross-section area **IS NOT** important*



$$SideExit(A_{ORF},A_0) := \left| \begin{array}{l} Ar \leftarrow \dfrac{\min(A_{ORF},A_0)}{\max(A_{ORF},A_0)} \\ 203.4 - 909.5\cdot Ar + 1105\cdot Ar^2 \ \text{ if } Ar \le 0.40 \\ 107.04 - 440.7\cdot Ar + 726.8\cdot Ar^2 - 539.87\cdot Ar^3 + 149.78\cdot Ar^4 \ \text{ if } Ar > 0.40 \end{array} \right.$$

Orifices - Thick Edge

*In terms of velocity at **ORIFICE** cross-section*
*Note: Order of entering cross-section area **IS** important*



$$\xi \equiv \frac{\Delta p}{\frac{1}{2}\rho\cdot V_0^{\,2}}$$

$$\xi_{ORF}(A_1, A_0, A_2, D_0, L, \lambda, Type) := \begin{array}{|l} r_L \leftarrow \dfrac{L}{D_0} \\ \xi_p \leftarrow \begin{array}{|ll} (2.4 - r_L)\cdot 10^{-\left(0.25+\frac{0.535\cdot r_L^{\,8}}{0.05+r_L^{\,8}}\right)} & \text{if Type = "Thick"} \\ 0.13 + 0.34\cdot 10^{-\left(3.45\cdot r_L+88.4\cdot r_L^{\,2.3}\right)} & \text{if Type = "Bevel"} \\ 0.50 & \text{if Type = "Sharp"} \end{array} \\ \tau_o \leftarrow \begin{array}{|ll} \xi_p & \text{if Type = "Thick"} \\ 2\sqrt{\xi_p} & \text{if Type = "Bevel"} \\ \sqrt{2} & \text{if Type = "Sharp"} \end{array} \\ \xi_o \leftarrow \begin{array}{|ll} 0.50 & \text{if Type = "Thick"} \vee \text{Type = "Sharp"} \\ \xi_p & \text{if Type = "Bevel"} \end{array} \\ \xi_o\cdot\left(1-\dfrac{A_0}{A_1}\right)^{0.75} + \tau_o\cdot\left(1-\dfrac{A_0}{A_2}\right)\cdot\left(1-\dfrac{A_0}{A_1}\right)^{0.375} + \left(1-\dfrac{A_0}{A_2}\right)^2 + \lambda\cdot\dfrac{L}{D_0} \end{array}$$

$$\xi_{ORF\_BVL}(A_1, A_0, A_2, D_0, L, \lambda) := \begin{array}{|l} \xi_p \leftarrow 0.13 + 0.34\cdot 10^{-\left[3.45\cdot\frac{L}{D_0}+88.4\cdot\left(\frac{L}{D_0}\right)^{2.3}\right]} \\ \xi_p\cdot\left(1-\dfrac{A_0}{A_1}\right)^{0.75} + 2\sqrt{\xi_p}\cdot\left(1-\dfrac{A_0}{A_2}\right)\cdot\left(1-\dfrac{A_0}{A_1}\right)^{0.375} + \left(1-\dfrac{A_0}{A_2}\right)^2 \end{array}$$

$$\xi_{SHARP}(A_1, A_0, A_2) := \left[\frac{1}{\sqrt{2}}\cdot\left(1-\frac{A_0}{A_1}\right)^{0.375} + \left(1-\frac{A_0}{A_2}\right)\right]^2$$

$$\xi_{ORF\_THK}(A_1, A_0, A_2, D_0, L, \lambda) := \begin{array}{|l} r_L \leftarrow \dfrac{L}{D_0} \\ \tau_o \leftarrow (2.4 - r_L)\cdot 10^{-\left(0.25+\frac{0.535\cdot r_L^{\,8}}{0.05+r_L^{\,8}}\right)} \\ \xi_o \leftarrow 0.50 \\ \xi_o\cdot\left(1-\dfrac{A_0}{A_1}\right)^{0.75} + \tau_o\cdot\left(1-\dfrac{A_0}{A_2}\right)\cdot\left(1-\dfrac{A_0}{A_1}\right)^{0.375} + \left(1-\dfrac{A_0}{A_2}\right)^2 + \lambda\cdot\dfrac{L}{D_0} \end{array}$$

$$\xi_{EPB} := \xi_{ORF\_THK}\left(49cm^2, 1.75cm^2, 49cm^2, D_H, 3.0mm, 0.02\right) = 2.08$$

$$\xi_{EPB2} := \xi_{ORF}\left(49cm^2, 1.75cm^2, 49cm^2, D_H, 3.0mm, 0.02, \text{"Thick"}\right) = 2.08$$